LAKSHYA

MHTCET 2025

Physics | Lecture - 06

Rotational Dynamics

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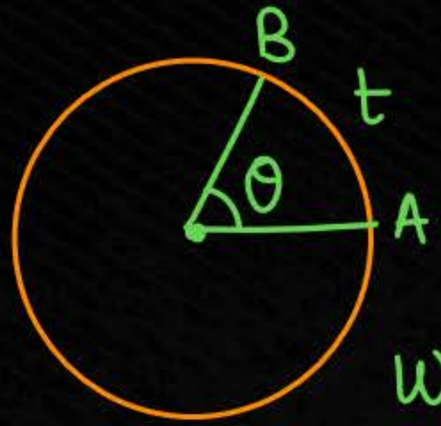
Topics to be covered

1. Moment of Inertia ✓ → for Boards
2. Radius of Gyration ✓
3. Theorem of Parallel Axis ✓ 7 → for Boards.
4. Theorem of Perpendicular Axis ✓ 6

Revision

-  Revol^n

 Rot^n

- 

$\omega = \frac{\theta}{t}$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

- $\alpha = \frac{\omega_2 - \omega_1}{t}$

- $\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

- $V = r\omega$

$\vec{V} = \vec{\omega} \times \vec{r}$

- $\vec{a} = \sqrt{a_t^2 + a_r^2}$

- $\vec{F}_{cp} = -\frac{mv^2}{r}\,\hat{r}$

- $V_{max} = \sqrt{\mu r g}$

- $V = \sqrt{r g \tan\theta}$

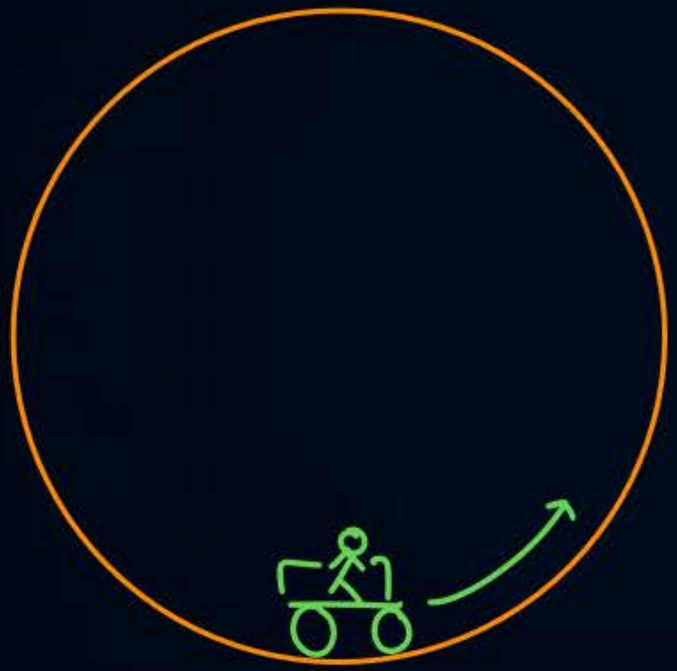
- V CM

$$v_{High} = \sqrt{rg}$$

$$v_{low} = \sqrt{5rg}$$

$$v_{mid} = \sqrt{3rg}.$$

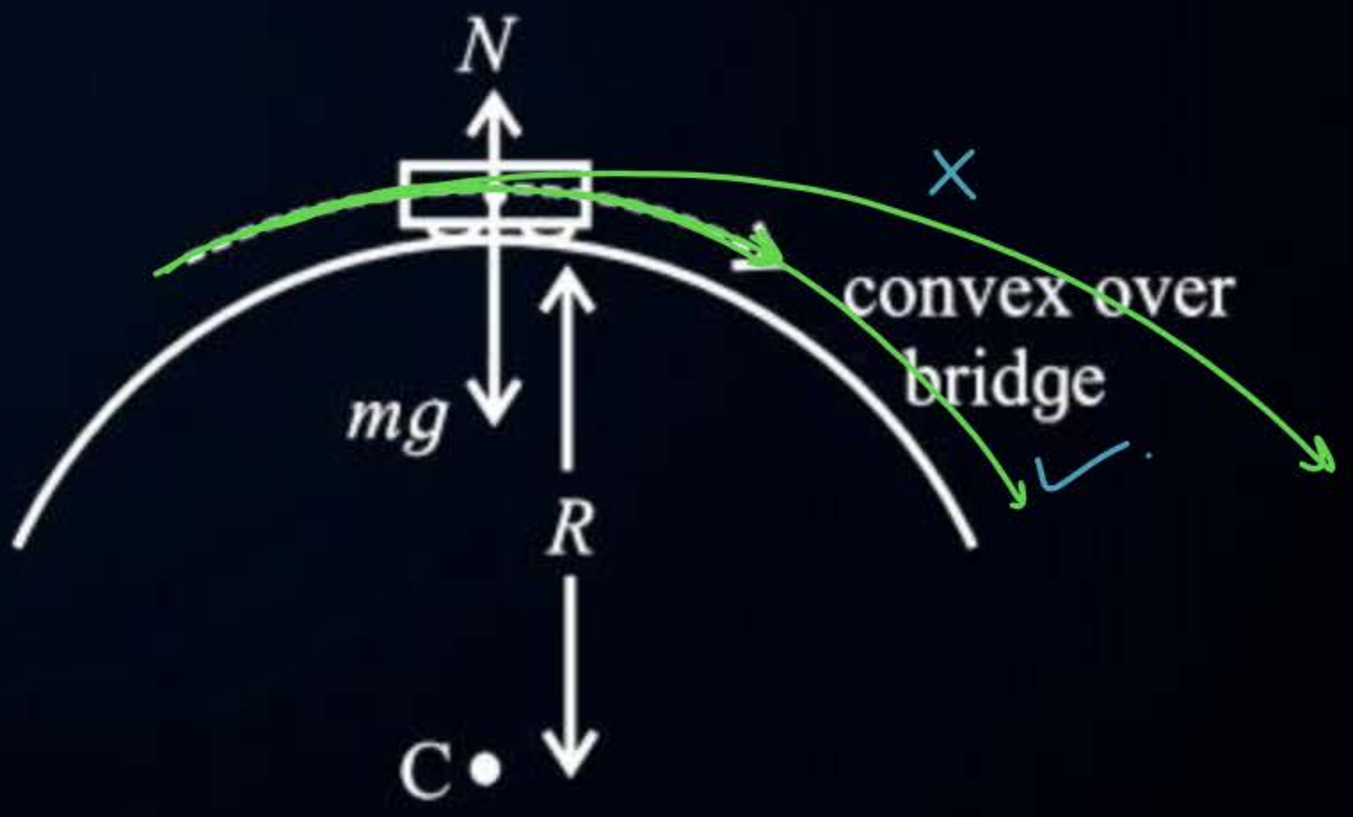
Sphere of Death (मृत्यु गोल)



- It is based on vertical circular motion as well as on horizontal circular motion.

Vehicle at the Top of a Convex Over-Bridge

- Maximum safe speed $= \sqrt{rg}$

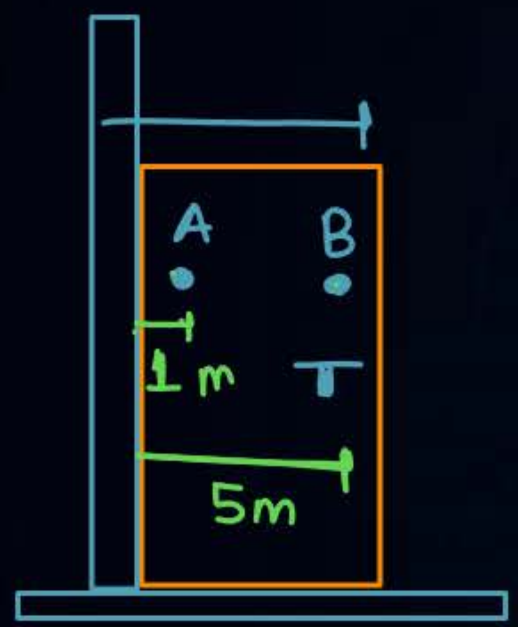


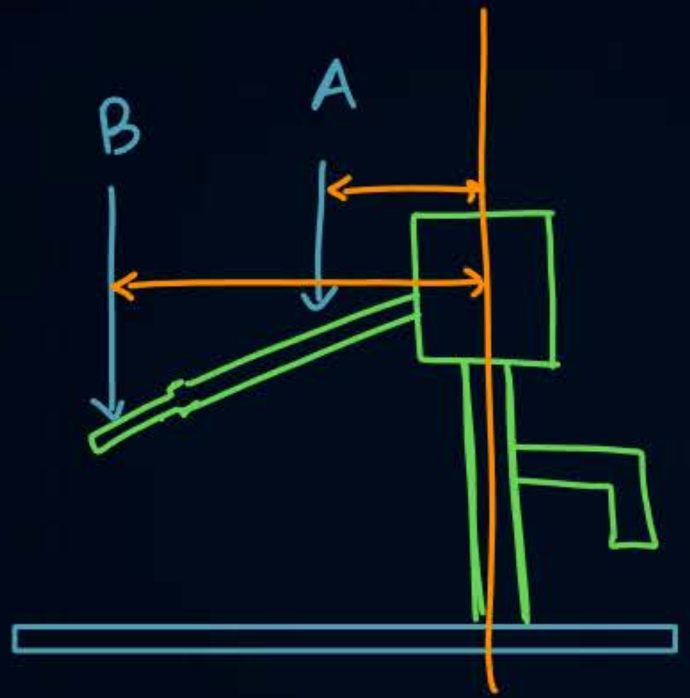
Vehicle on a convex over-bridge

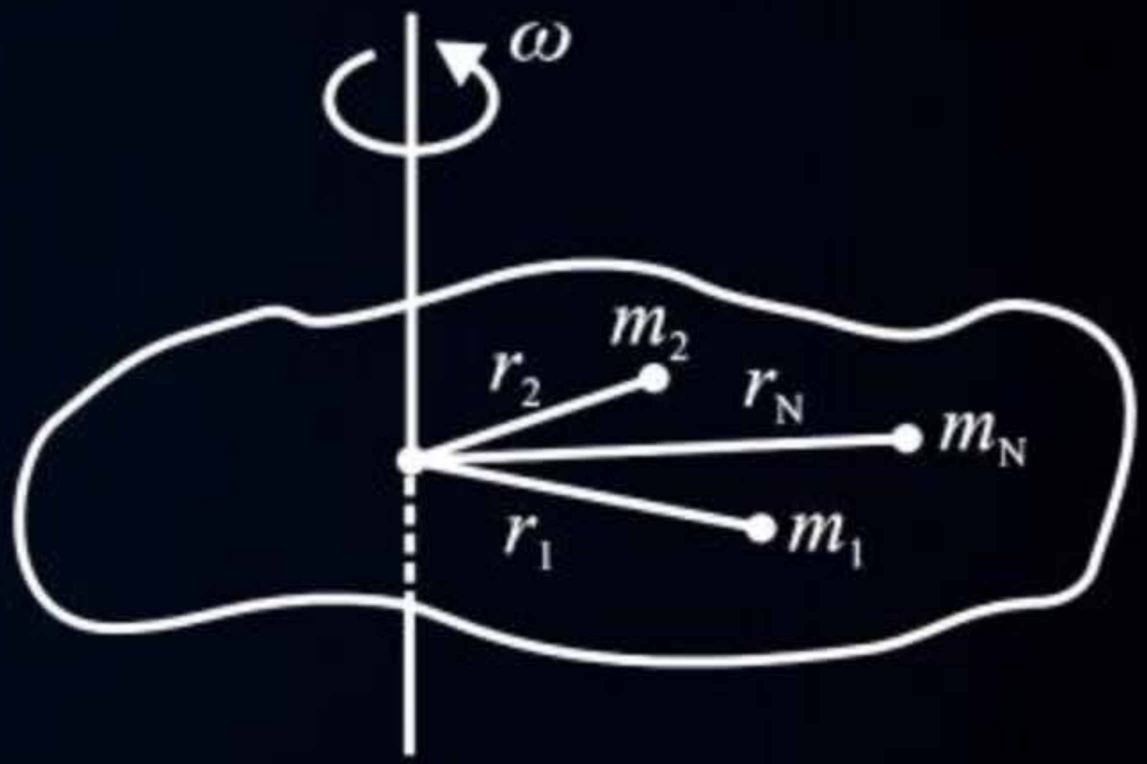
Moment of Inertia as an Analogous Quantity for Mass

$\tau_A = 1$

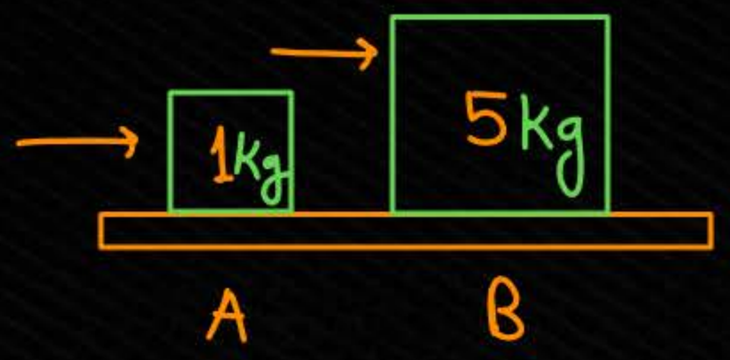
$\tau_B = 5$

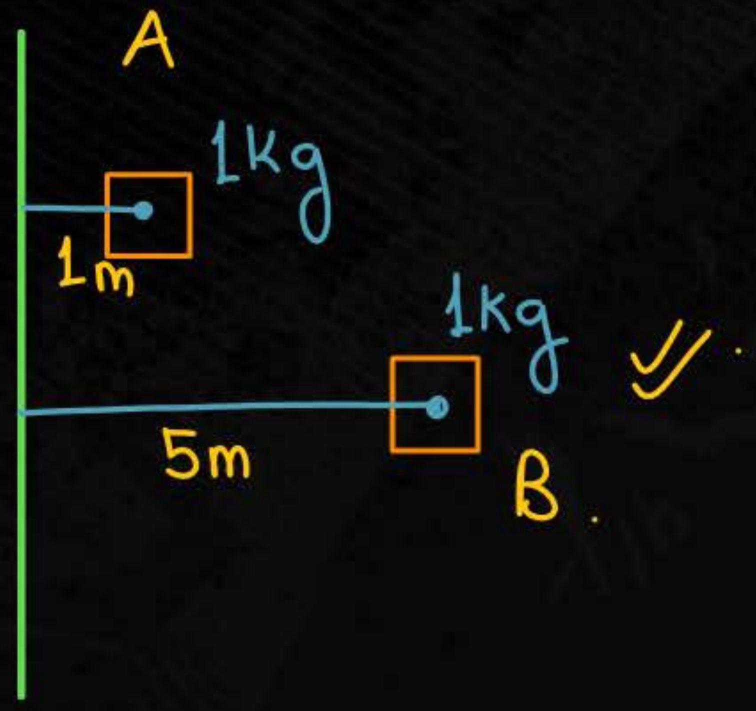






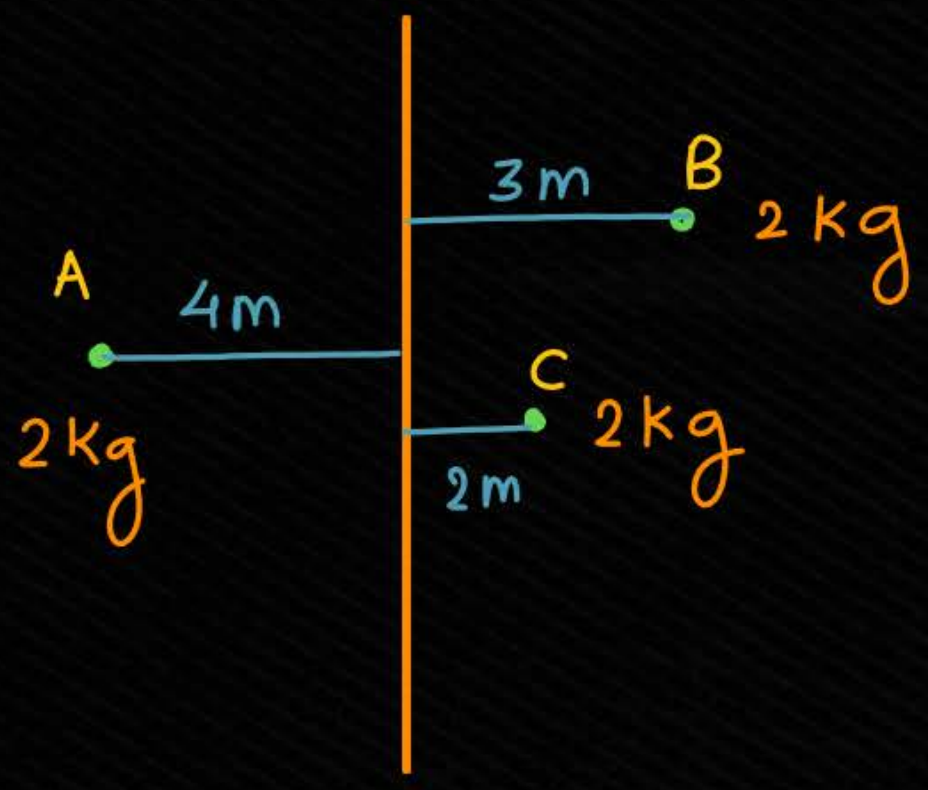
- For rotating any object not only force but also distance from axis of rotation where force is applied is also important. & that combined quantity is c/a Torque

- 



$$\boxed{I = mr^2}$$

Que:



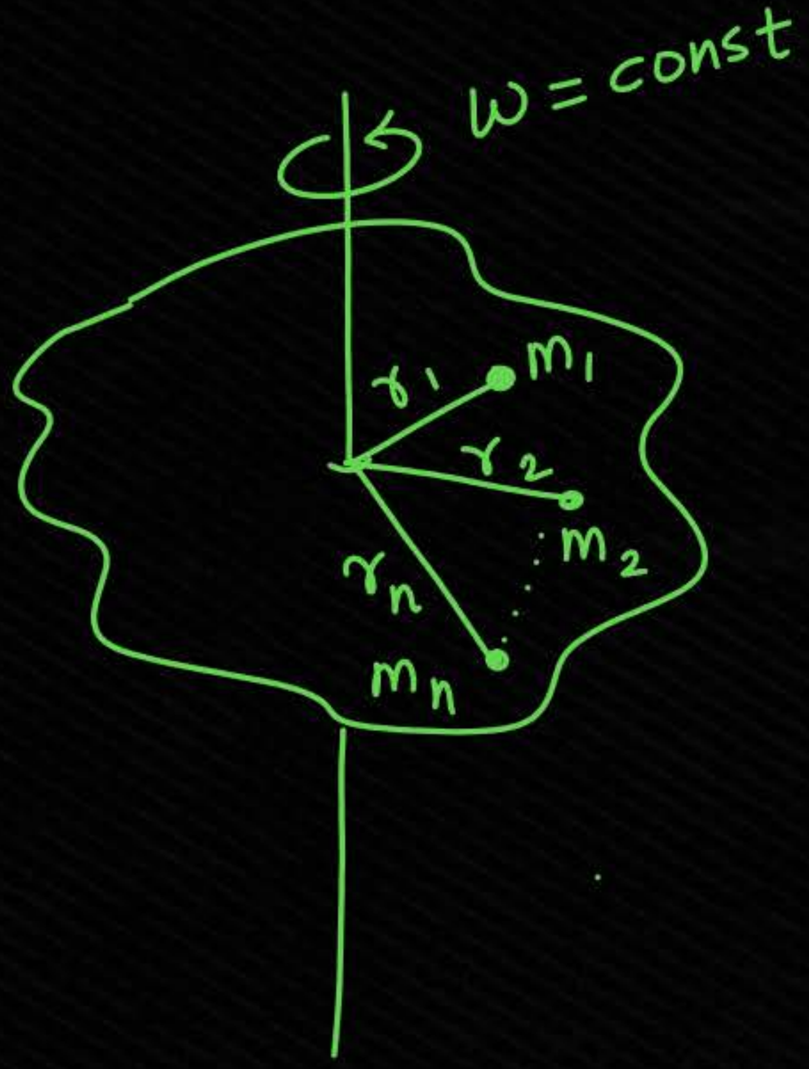
Calculate M of I of A, B, & C

$$I_A = m_A r_A^2$$
$$= 2 \times 4^2$$
$$= 2 \times 16$$
$$I_A = 32\ kgm^2$$

$$I_B = 18\ kgm^2$$

$$I_C = 8\ kgm^2$$

Derivation for M of I :



$(v = r\omega)$

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2$$

$$K.E_{rot} = K.E_1 + K.E_2 + \dots + K.E_n$$

$$= \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \frac{1}{2}m_nr_n^2\omega^2$$

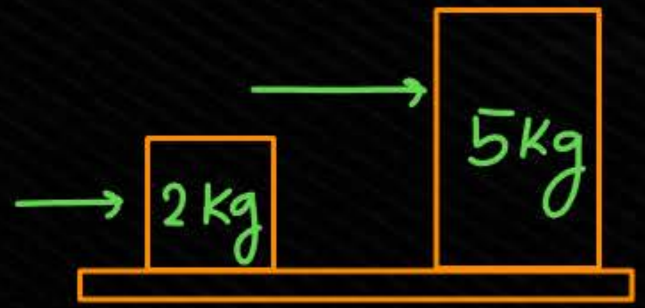
$$K.E_{rot} = \frac{1}{2}\omega^2\left[m_1r_1^2 + m_2r_2^2 + \dots + m_nr_n^2\right]$$

$$K.E = \frac{1}{2}mv^2$$

$$K.E_{rot} = \frac{1}{2} \underline{\underline{I}} \omega^2$$

$$I = \left[m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_n r_n^2 \right]$$

Linear Mot^n



$\boxed{\text{force} \propto \text{Mass.}}$

- Here mass opposes motion

Rotational Mot^n

$I = mr^2$

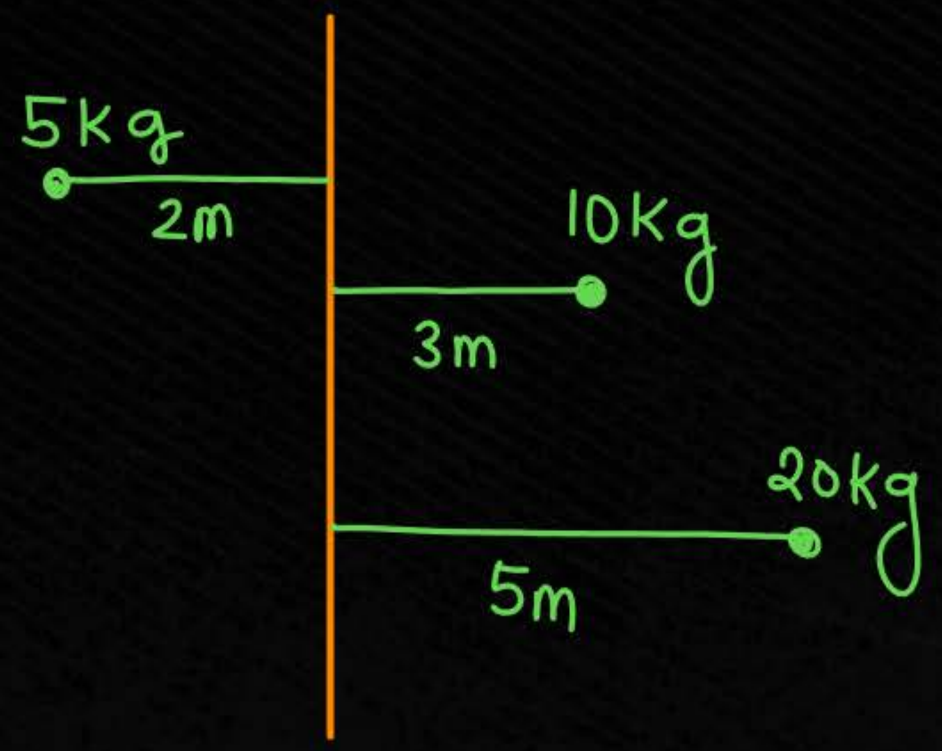
$\boxed{\text{Torque} \propto \text{M of I.}}$

- Here M of I opposes motion.

Que :

Calculate M of I

of following system



$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= 5 \times 4 + 10 \times 9 + 20 \times 25$$

$$= 20 + 90 + 500$$

$$I = 610 \, kgm^2.$$

Radius of Gyration

$$\cancel{M}K^2 = \frac{\cancel{M}R^2}{2}$$

$$\boxed{K = \frac{R}{\sqrt{2}}}$$

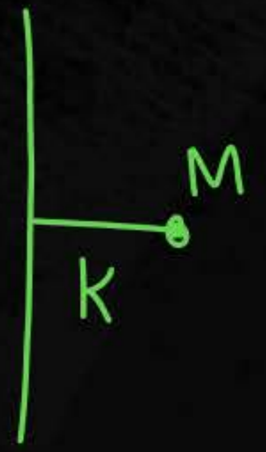
$$\boxed{I = MK^2}$$

$$\boxed{I = \frac{MR^2}{2}}$$

$\underline{\text{Def}^n}$:

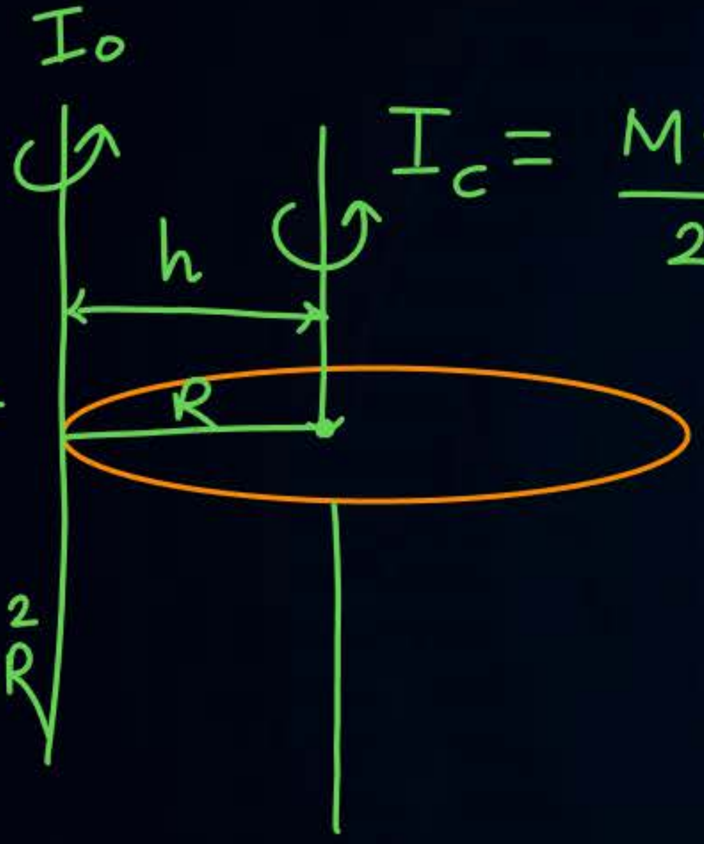
It is the distance from axis of rotation upto a point where whole mass of body is supposed to be concentrated and that will give same M of I as that of object

$I = MK^2$



$I = \frac{MR^2}{2}$

Theorem of Parallel Axes

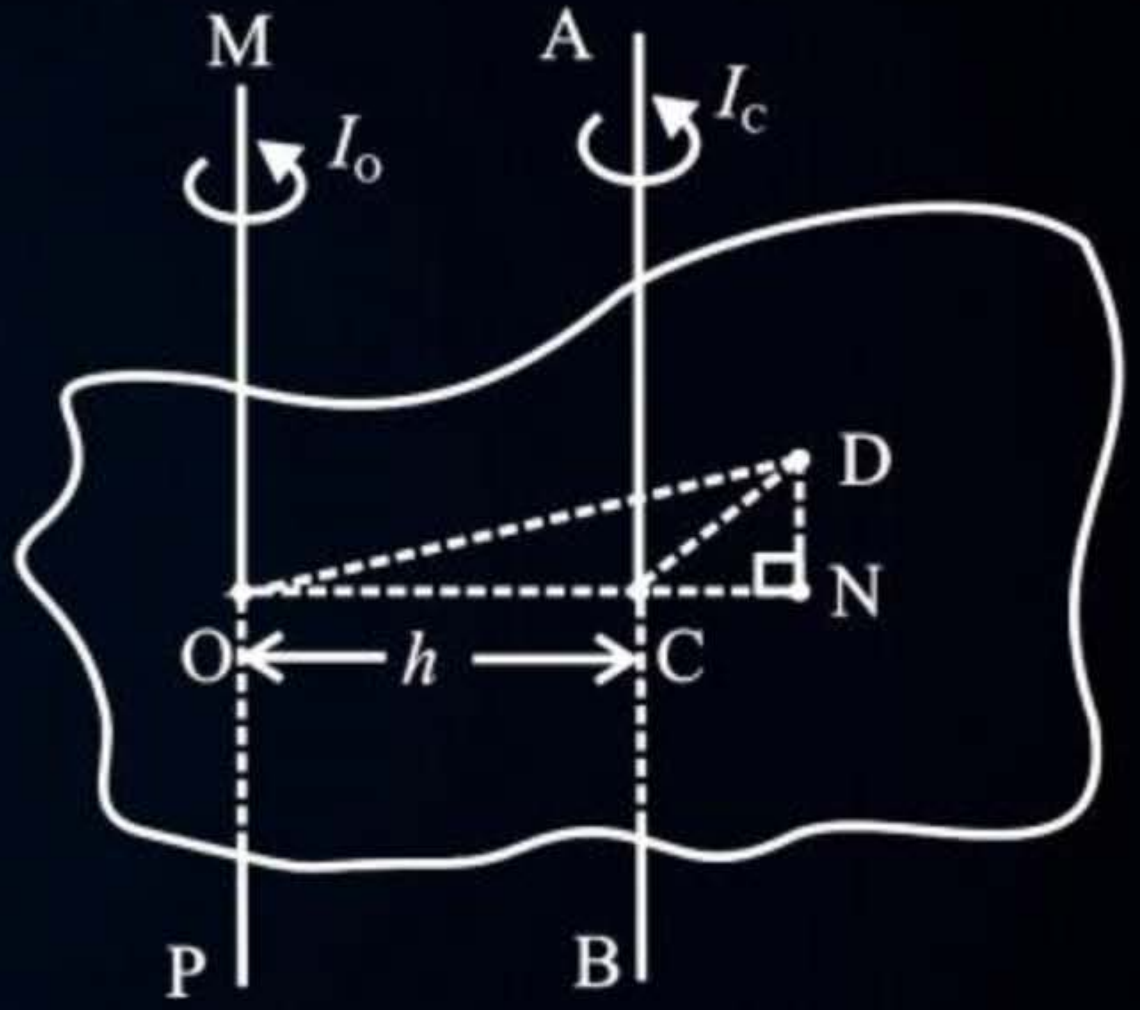


$I_c = \frac{MR^2}{2}$

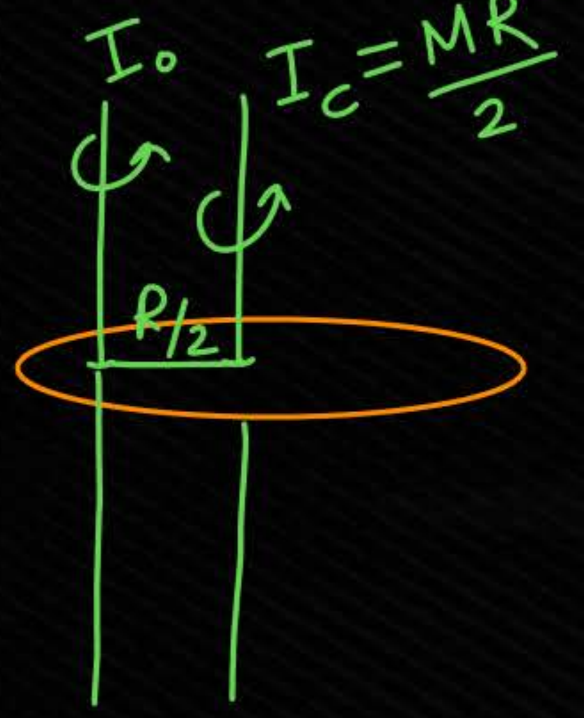
$I_0 = I_c + Mh^2$

$I_0 = \frac{MR^2}{2} + MR^2$

$I_0 = \frac{3}{2}MR^2$



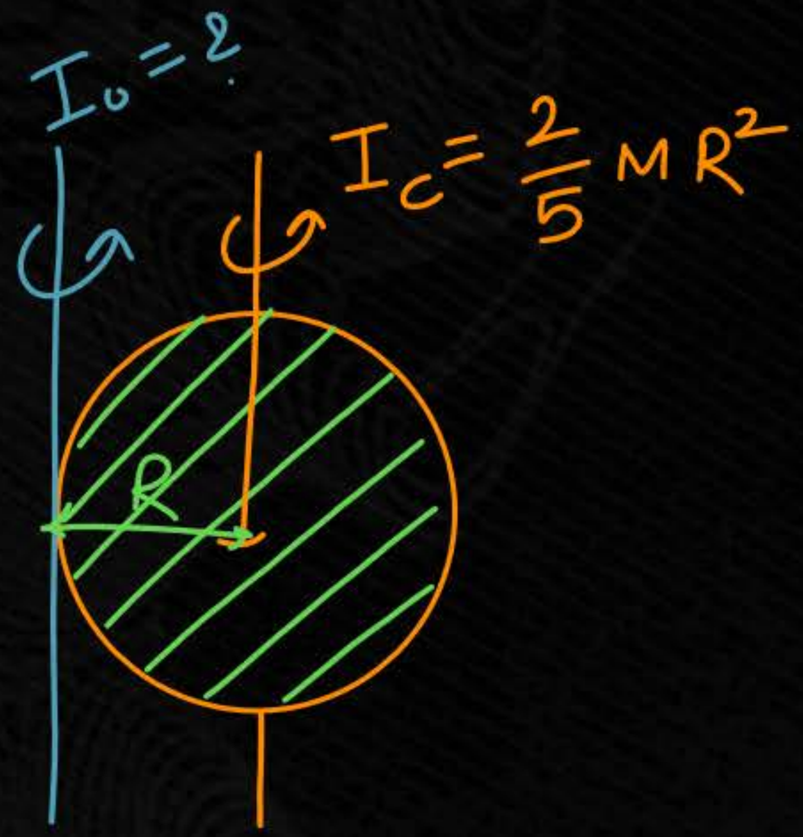
Theorem of parallel axes



$$I_0 = I_c + Mh^2$$

$$= \frac{MR^2}{2} + \frac{MR^2}{4}.$$

$$I_0 = \frac{3}{4}MR^2$$

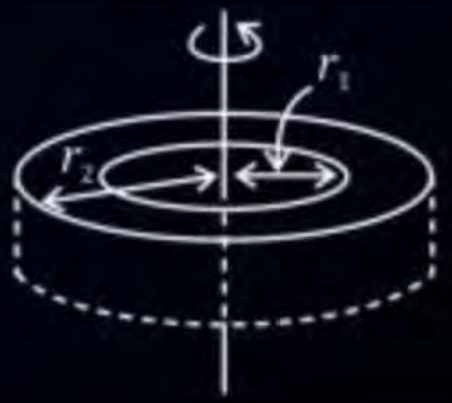


$$I_0 = I_c + Mh^2$$

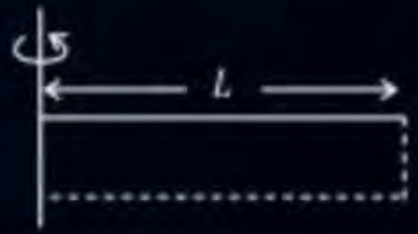
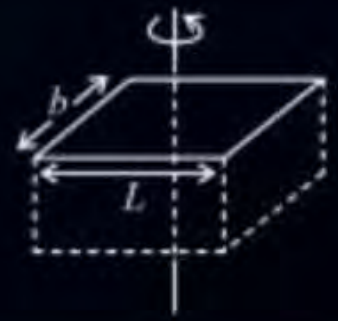
$$= \frac{2}{5}MR^2 + MR^2$$

$$I_0 = 7/5\,MR^2$$

Table 3: Expressions for moment of inertias for some symmetric objects:

Object	Axis	Expression of Moment of inertia	Figure
Thin ring or hollow cylinder	Central	$I = MR^2$	
Thin ring	Diameter	$I = \frac{1}{2}MR^2$	
Annular ring or thick walled hollow cylinder	Central	$I = \frac{1}{2}M(r_2^2 + r_1^2)$	

Object	Axis	Expression of Moment of inertia	Figure
Uniform disc or Solid cylinder	Central	$I = \frac{1}{2}MR^2$	
Uniform disc	Diameter	$I = \frac{1}{4}MR^2$	
Thin walled hollow sphere	Central	$I = \frac{2}{3}MR^2$	
Solid spere	Central	$I = \frac{2}{5}MR^2$	

Object	Axis	Expression of Moment of inertia	Figure
Uniform symmetric	Central	$I = \frac{2}{5} M \frac{(r_2^5 - r_1^5)}{(r_2^3 - r_1^3)}$	
Thin uniform rod or rectangular plate	Perpendicular to length and passing through centre	$I = \frac{1}{12} ML^2$	
Thin uniform rod or rectangular plate	Perpendicular to length and about one end	$I = \frac{1}{3} MR^2$	
Uniform plate or rectangular parallelepiped	Central	$I = \frac{1}{12} M(L^2 + b^2)$	

Object	Axis	Expression of Moment of inertia	Figure
Uniform solid right circular cone	Central	$I = \frac{3}{10} MR^2$	
Uniform hollow right circular cone	Central	$I = \frac{1}{2} MR^2$	

Summary

1) Moment of Inertia $I = \sum_{i=1}^{n} m_i r_i^2$

$$I = MR^2 = \int r^2 dm.$$

2) Radius of Gyration (k) : $I = Mk^2$

$$k = \sqrt{\frac{I}{M}}.$$

Homework

1) Memorize defn of parallel Axis theorem

2) Memorize formula for M of I of Disc, Ring, solid sphere & Rod.

3) Revise lecture

धन्यवाद