LAKSHYA

MHTCET 2025

Physics | Lecture - 08

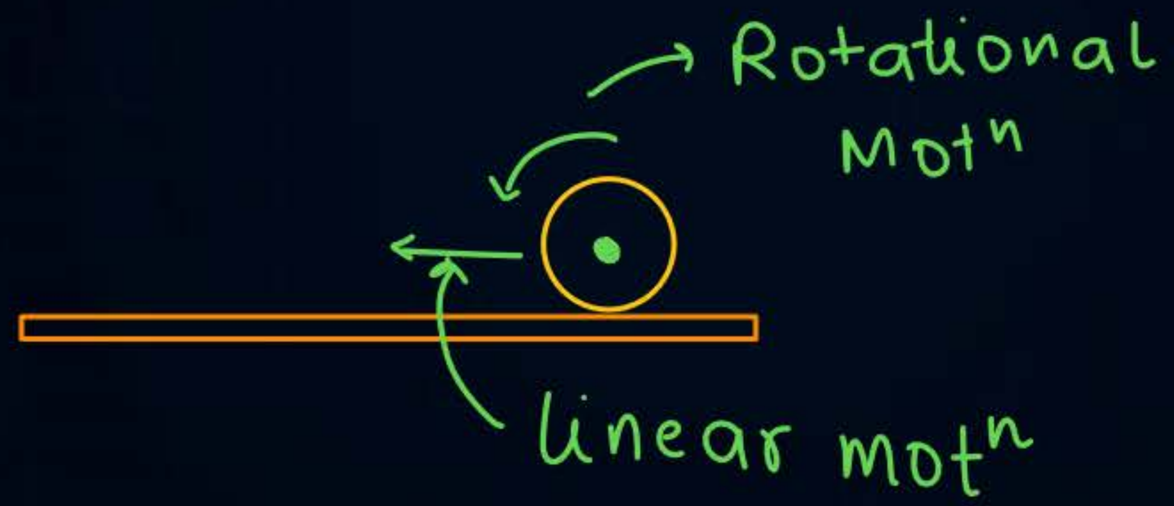
Rotational Dynamics

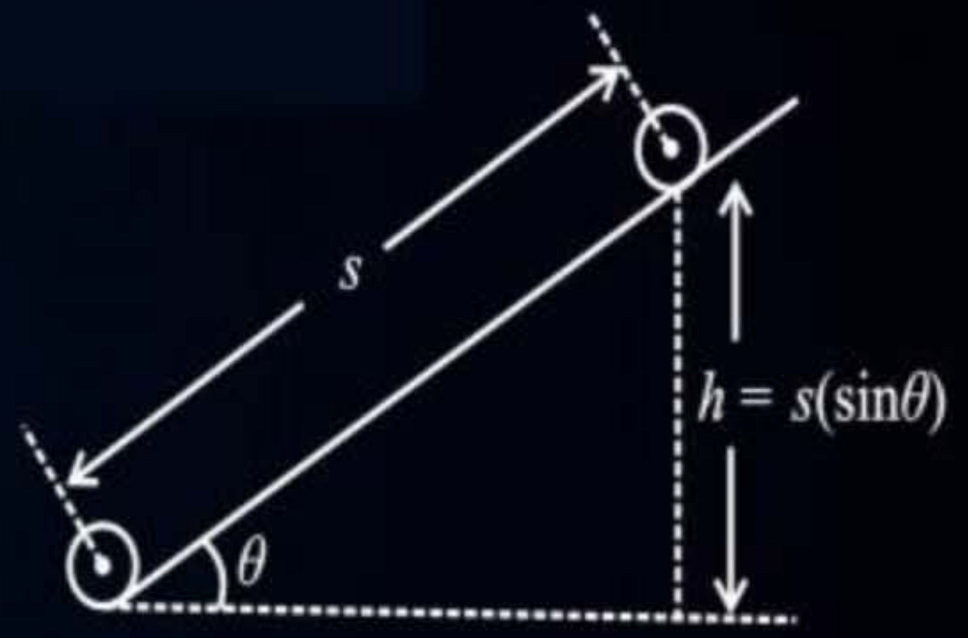
By - Sushant Sir

Topics *to be covered*

1. Rolling Motion ✓

Linear Acceleration and Speed While Pure Rolling Down an Inclined Plane





Rolling along an incline

As body rolls along horizontal surface it performs two motions

1) Rotational Motion — body

2) Linear/ Translational Motion — Center of Mass.

$\therefore$ Total Energy $= K.E_{rot} + K.E_{trans.}$

$$= \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2$$

$$\therefore T.E = \frac{1}{2} MK^2 \frac{V^2}{R^2} + \frac{1}{2} MV^2.$$

$$\boxed{T.E = \frac{1}{2} MV^2 \left[1 + \frac{K^2}{R^2}\right]}$$

— Total Energy of Rolling Body.

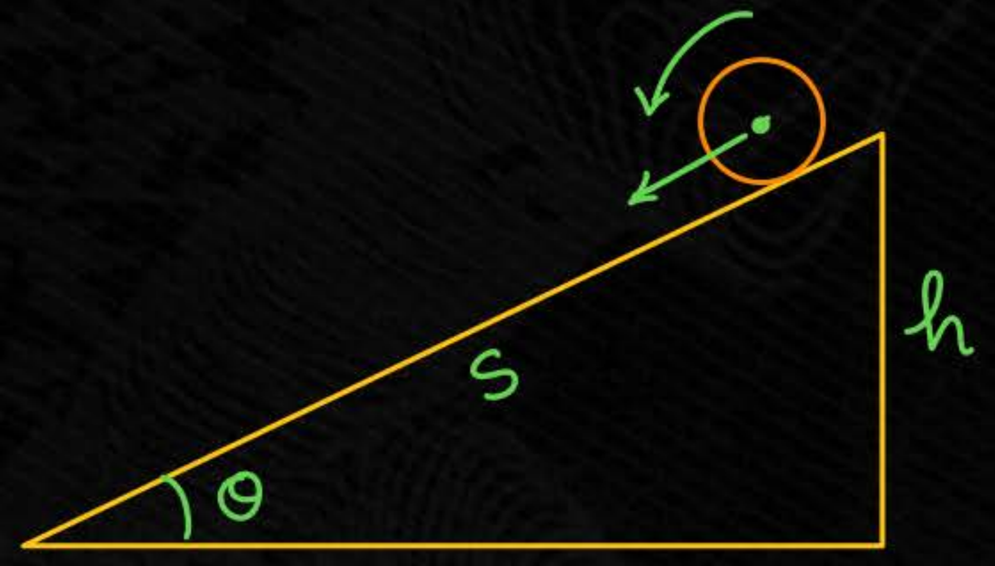
Potential Energy at height h

is mgh.

when object is released its

P.E is converted to K.E

$$\therefore \cancel{m}gh = \frac{1}{2}\cancel{m}v^2\left[1+\frac{K^2}{R^2}\right]$$



$$\sin\theta = \frac{h}{s}$$

$$s = \frac{h}{\sin\theta}$$

$$\therefore \quad \boxed{v = \sqrt{\frac{2gh}{\left[1+\frac{K^2}{R^2}\right]}}}$$

— Eqn for velocity.

$$\therefore \quad \boxed{v^2 = u^2 + 2as}$$

$$\frac{\cancel{2}g\cancel{h}}{\left[1+\frac{K^2}{R^2}\right]} = 0 + \cancel{2}a\frac{\cancel{h}}{\sin\theta}$$

$$\boxed{a = \frac{g\sin\theta}{\left[1+\frac{K^2}{R^2}\right]}}$$

— Eqn for acc^n

Table 1 : Analogous kinematical equations (ω_0 is the initial angular velocity)

Equation for translational motion	**Analogous equation for rotational motion**
$v_{av} = \frac{u+v}{2}$	$\omega_{av} = \frac{\omega_0+\omega}{2}$
$a = \frac{dv}{dt} = \frac{v-u}{t}$ $\therefore v = u + at$	$a = \frac{d\omega}{dt} = \frac{\omega-\omega_0}{t}$ $\therefore \omega = \omega_0 + \alpha t$
$s = v_{av} . t$ $= \left(\frac{u+v}{2}\right) t$ $= ut + \frac{1}{2} at^2$	$\theta = \omega_{av} . t$ $= \left(\frac{\omega_0+\omega}{2}\right) t$ $= \omega_0 t + at^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2a\theta$

Table 2 : Analogous quantities between translational motion and rotational motion

Translational motion		Rotational motion		
Quantity	Symbol/ expression	Quantity	Symbol/ expression	Inter-relation, if possible
Linear displacement	$\vec{s}$	Angular displacement	$\vec{\theta}$	$\vec{s} = \vec{\theta} \times \vec{r}$
Linear velocity	$\vec{v} = \frac{d\vec{s}}{dt}$	Angular velocity	$\vec{\omega} = \frac{d\vec{\theta}}{dt}$	$\vec{v} = \vec{\omega} \times \vec{r}$
Linear acceleration	$\vec{a} = \frac{d\vec{v}}{dt}$	Angular acceleration	$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$	$\vec{\alpha} = \vec{a} \times \vec{r}$
Inertia or mass	M	Rotational inertia or moment of inertia	I	$I = \int r^2 dm = \sum m_i r_i^2$

Table 2 : Analogous quantities between translational motion and rotational motion:

Translational motion		Rotational motion		
Quantity	Symbol/ expression	Quantity	Symbol/ expression	Inter-relation, if possible
Linear momentum	$\vec{p} = m\,\vec{v}$	Angular momentum	$\vec{L} = I\vec{\omega}$	$\vec{L} = \vec{r} \times \vec{p}$
Force	$\vec{f} = \frac{d\vec{p}}{dt}$	Torque	$\vec{\tau} = \frac{d\vec{L}}{dt}$	$\vec{\tau} = \vec{r} \times \vec{f}$
Work	$W = \vec{f}\,.\,\vec{s}$	Work	$W = \vec{\tau}\,.\,\vec{\theta}$	______
Power	$P = \frac{dW}{dt} = \vec{f}.\vec{v}$	Power	$P = \frac{dW}{dt} = \vec{\tau}.\vec{\omega}$	______

QUESTION

A lawn roller of mass 80 kg, radius 0.3 m and moment of inertia 3.6 kg.m^2, is drawn along a level surface at a constant speed of 1.8 m/s. Find (i) the translational kinetic energy (ii) the rotational kinetic energy (iii) the total kinetic energy of the roller.

$M = 80\,kg$

$R = 0.3\,m$

$I = 3.6\,kgm^2$

$V = 1.8\,m/s$

$K.E_{tran} = \frac{1}{2} M V^2$

$= \frac{1}{\cancel{2}} \times \overset{40}{\cancel{80}} \times (18 \times 10^{-1})^2$

$= 40 \times 324 \times 10^{-2}$

$= 12960 \times 10^{-2}$

$\boxed{K.E_{trans.} = 129.6\,J}$

$$K.E_{rot} = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} I \frac{V^2}{R^2}$$

$$= \frac{1}{\cancel{2}} \times \overset{1.8}{\cancel{3.6}} \times \frac{(18 \times 10^{-1})^2}{(3 \times 10^{-1})^2}$$

$$= \frac{\overset{0.2}{\cancel{1.8}} \times 324 \times \cancel{10^{-2}}}{\cancel{9} \times \cancel{10^{-2}}}$$

$$\boxed{K.E_{rot} = 64.8\,J}$$

$$T.E = K.E_{trans} + K.E_{rot}$$

$$= 129.6 + 64.8$$

$$\boxed{T.E = 194.4\,J}$$

QUESTION

A solid sphere of mass 1 kg rolls on a table with linear speed 2 m/s, find its total kinetic energy.

(A) 2.8 J

(B) 2.3 J

(C) 2 J

(D) 3 J

$M = 1\,kg$

$V = 2\,m/s$

$T.E = ?$

$T.E = \frac{1}{2}MV^2\left[1+\frac{K^2}{R^2}\right]$

$I_s = \frac{2}{5}MR^2$

$\cancel{M}K_s^2 = \frac{2}{5}\cancel{M}R^2$

$\frac{K_s^2}{R^2} = \frac{2}{5}$

$T.E = \frac{1}{\cancel{2}}\times 1\times \cancel{4}^{2}\left[1+\frac{2}{5}\right]$

$= \frac{7}{5}\times 2$

$= 1.4\times 2$

$\boxed{T.E = 2.8\,J}$

QUESTION

A ring and a disc having the same mass roll on a horizontal surface without slipping with the same linear velocity. If the total KE of the ring is 8 J, what is the total KE of the disc?

(A) 6 J

(B) 5 J

(C) 7 J

(D) 4 J

$M_{ring} = M_{Disc} = M$

$V_{ring} = V_{Disc} = V$

$T.E_{ring} = 8J$

$T.E_{Disc} = ?$

$T.E = \frac{1}{2}MV^2\left[1 + \frac{K^2}{R^2}\right]$

$I_{ring} = MR^2 = MK^2$

$K^2_{Ring} = R^2_{Ring}$

$T.E_{Ring} = MV^2$

$$\therefore \boxed{8 = MV^2}$$

$$T.E_{Disc} = \frac{1}{2}MV^2\left[1+\frac{K^2}{R^2}\right]$$

$$\frac{\cancel{M}R^2}{2} = \cancel{M}K^2$$

$$\frac{K^2}{R^2} = \frac{1}{2}$$

$$T.E = \frac{1}{2}MV^2\left[\frac{3}{2}\right]$$

$$T.E_{Disc} = \frac{3}{4}MV^2$$

$$= \frac{3}{4} \times 8$$

$$\boxed{T.E_{Disc} = 6J}$$

Summary

1) Rolling Motion. $T.E = \frac{1}{2}MV^2\left[1+\frac{K^2}{R^2}\right]$

Homework

1) Revise all Notes

2) Solve all numericals Multiple times.

3) Read text book.

Power :

$$x = 100 = 10^{2}$$

$$x = 10^{-2} = \frac{1}{100}$$


धन्यवाद