LAKSHYA

MHTCET 2025

Physics | Lecture - 07

Rotational Dynamics

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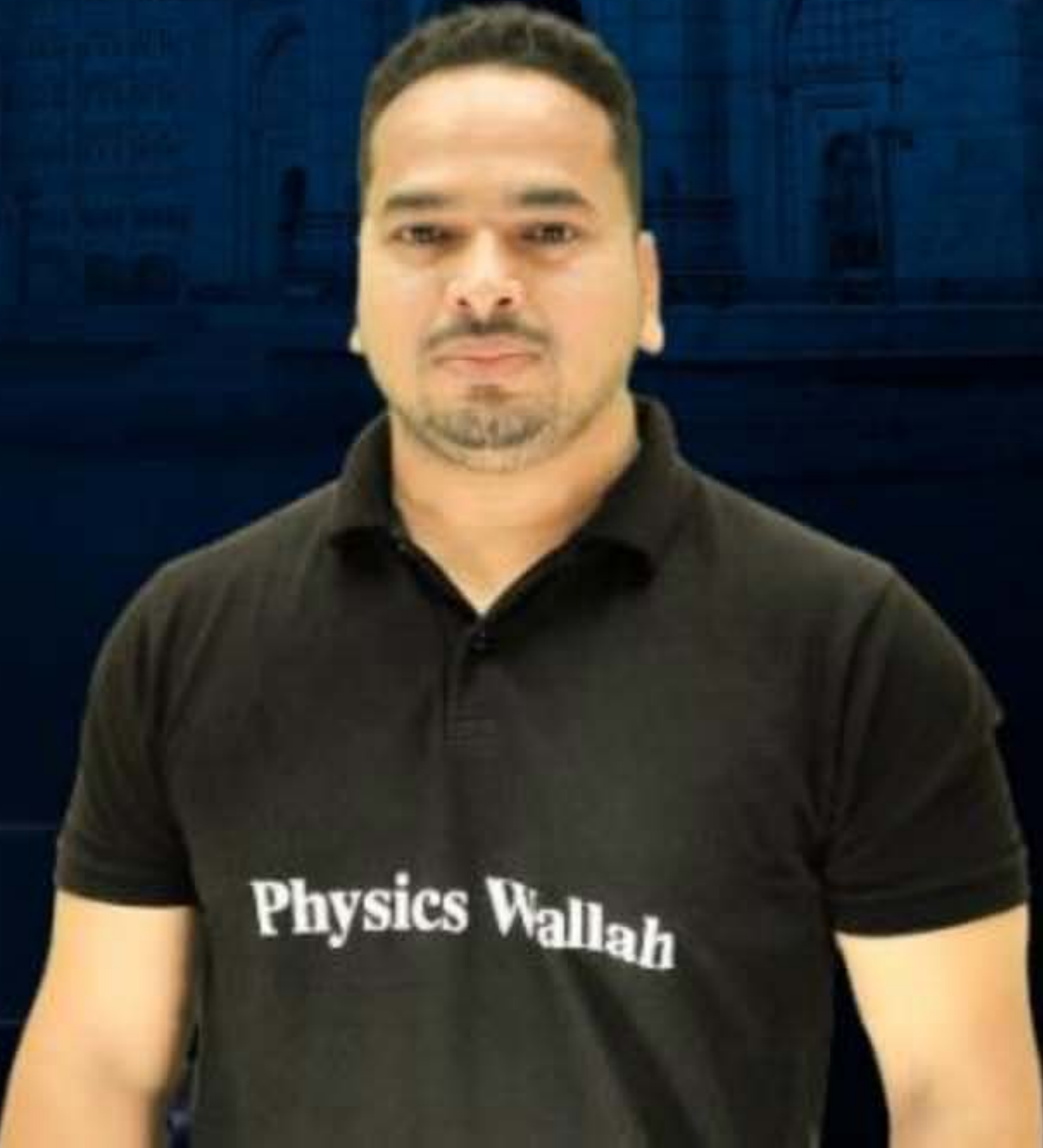


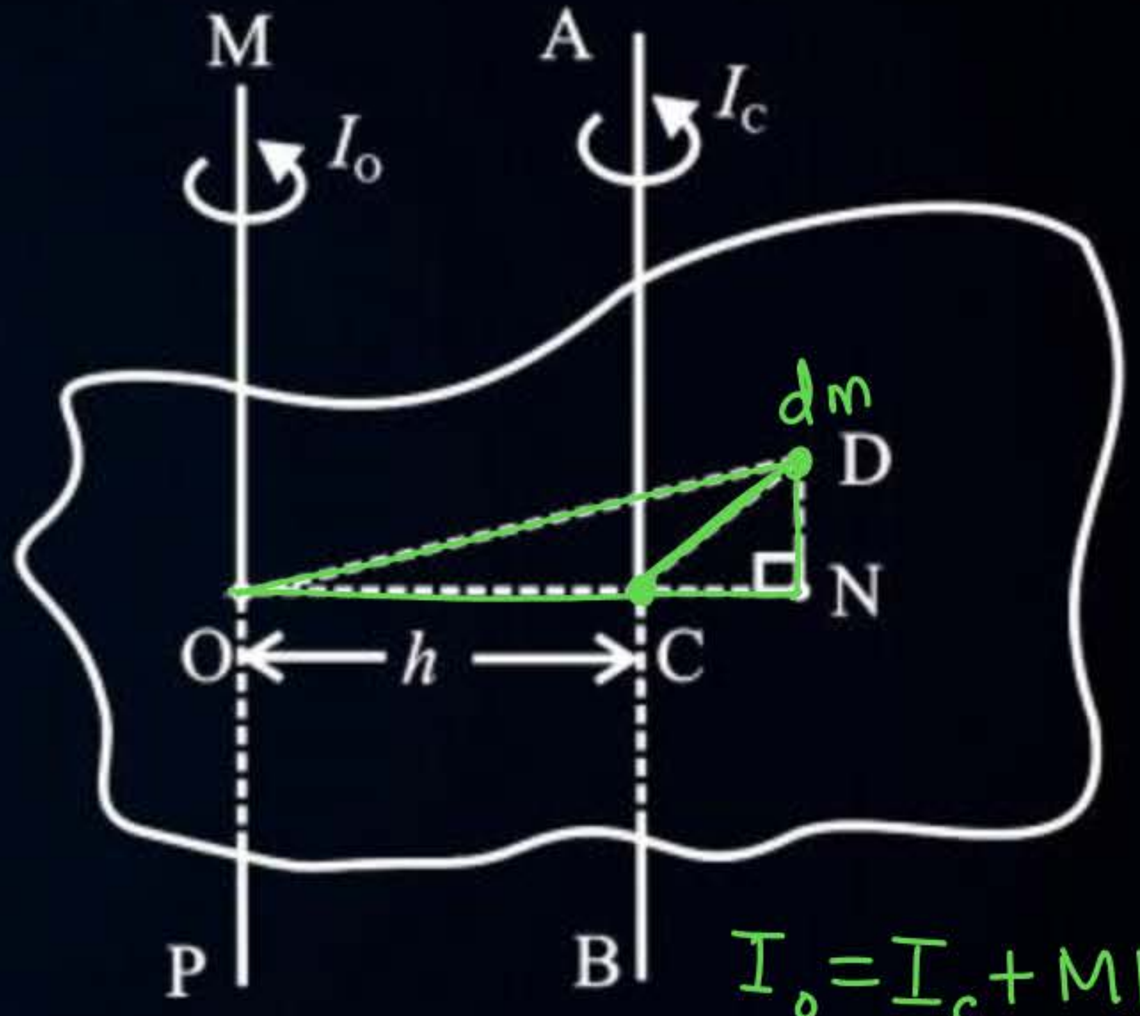
Topics *to be covered*

1. Theorem of Parallel Axis ✓
2. Theorem of Perpendicular Axis ✓
3. Angular Momentum and Its Conservation ✓
4. Torque in terms of Moment of Inertia ✓

Theorem of Parallel Axes

- $I = \int r^2 \, dm.$
- $I_c = \int (CD)^2 \, dm.$
- $I_0 = \int (OD)^2 \, dm.$

from fig $OD^2 = DN^2 + NO^2$



$I_0 = I_c + Mh^2$

Theorem of parallel axes

$$\therefore I_0 = \int (DN^2 + NO^2)\, dm.$$

$$= \int (DN^2 + (NC + CO)^2)\, dm.$$

$$= \int [\underline{DN}^2 + \underline{NC}^2 + CO^2 + 2 \cdot NC \cdot CO]\, dm.$$

$$= \int [CD^2 + CO^2 + 2NC \cdot CO]\, dm.$$

$$= \int CD^2\, dm + \int CO^2\, dm + \int 2NC \cdot CO\, dm$$

$$I_0 = I_c + \int h^2\, dm + \int 2NC \cdot h\, dm.$$

$$I_0 = I_c + Mh^2 + 0$$

$$\therefore \boxed{I_0 = I_c + Mh^2}$$

Theorem of Perpendicular Axes

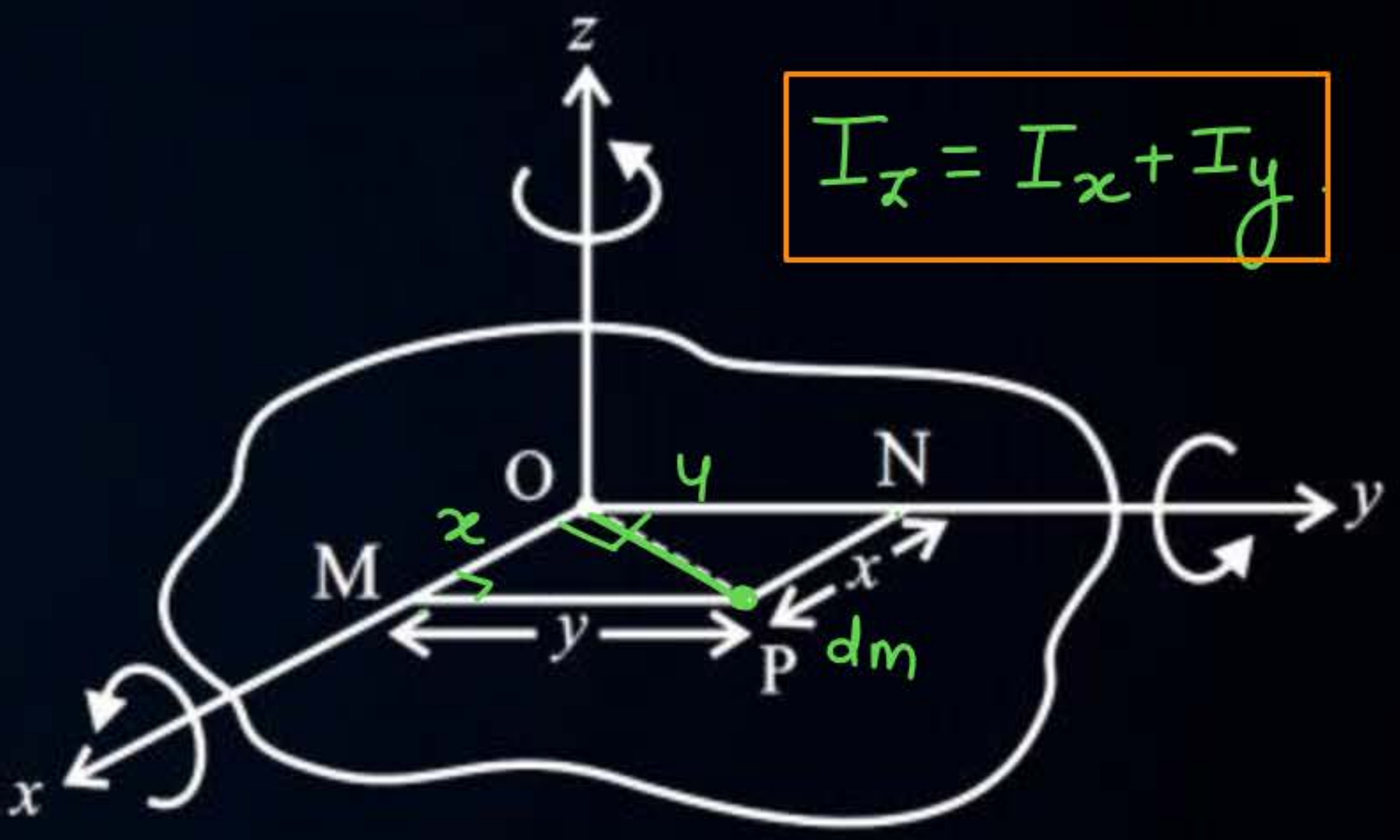
$$OP^2 = x^2 + y^2$$

$$OP = \sqrt{x^2 + y^2}$$

$$I_x = \int y^2\,dm \qquad I_y = \int x^2\,dm$$

$$I_x + I_y = \int (x^2 + y^2)\,dm$$

$$\therefore I_x + I_y = \int z^2\,dm = I_z$$

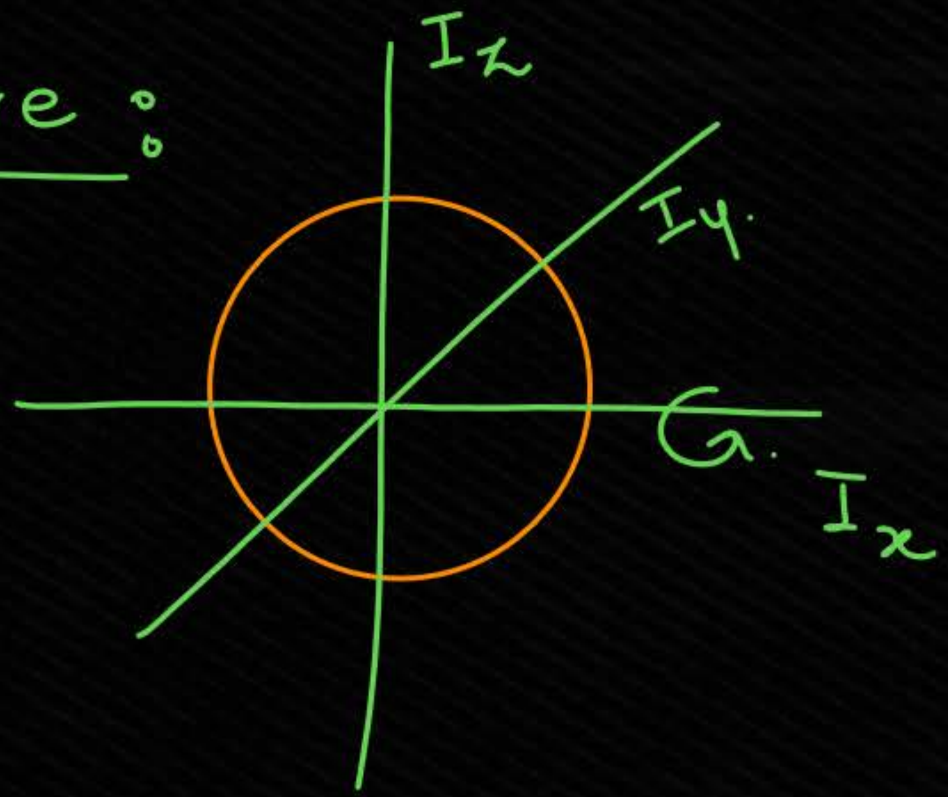


$$I_z = I_x + I_y$$

Theorem of perpendicular axes

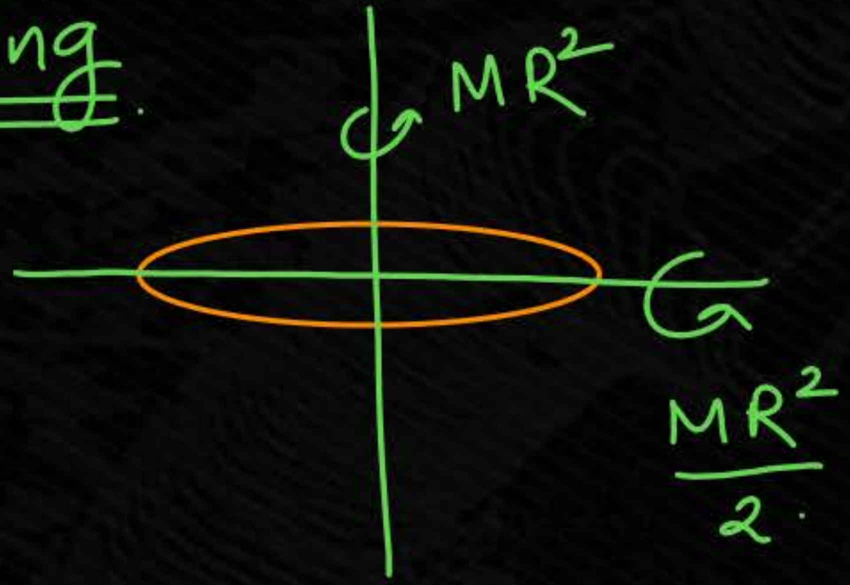
Que: find M of I

Sphere :



$$I_z = \frac{2}{5} MR^2$$

Ring.



Angular Momentum or Moment of Linear Momentum

$$\vec{P} = m\vec{v}$$

$v = r\omega$

$v_1 = r_1\omega$

$v_2 = r_2\omega$

$v_3 = r_3\omega$

$$\vec{L} = \vec{r} \times \vec{p}$$

$L = rp$

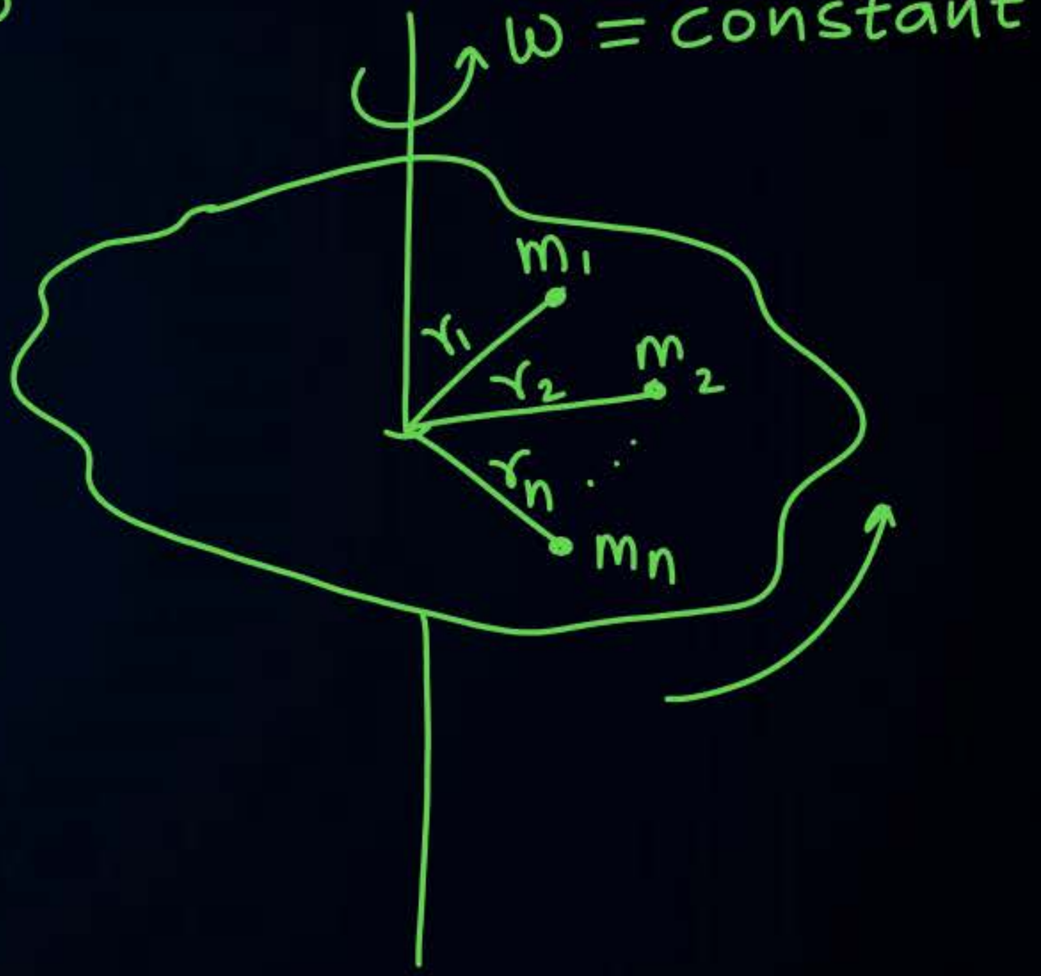
$L = rmv$

$L_1 = r_1 m_1 r_1 \omega$

$L_1 = m_1 r_1^2 \omega$

$L_2 = m_2 r_2^2 \omega$

$L_n = m_n r_n^2 \omega$



$$L = L_1 + L_2 + \dots + L_n$$

$$= m_1 r_1^2 \omega + m_2 r_2^2 \omega + \dots + m_n r_n^2 \omega$$

$$= (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2)\, \omega$$

$$\boxed{L = I \omega}$$

Conservation of Angular Momentum

So long as external torque acts on an object is zero, angular momentum is conserved.

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \vec{p} \times \frac{d\vec{r}}{dt}$$

$$= \vec{r} \times \vec{f} + \vec{p} \times \vec{v}$$

$$\therefore \frac{d\vec{L}}{dt} = \vec{\tau} + m\vec{v} \times \vec{v}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

$$\therefore \text{if } \tau = 0 \quad \frac{d\vec{L}}{dt} = 0$$

hence angular momentum is conserved.

$$\vec{L}_1 = \vec{L}_2 \quad \text{if } \tau = 0$$

$$I_1\omega_1 = I_2\omega_2.$$

I don't ignore anyone's chat all are my cute little brother's & sister's

Sushant.

Expression for Torque in Terms of Moment of Inertia

$$\vec{\tau} = \vec{r} \times \vec{f}$$

$$= \vec{r} \times m\vec{a}$$

$$= \vec{r} \times m\vec{r} \times \vec{\alpha}.$$

$$\tau = mr^2\alpha$$

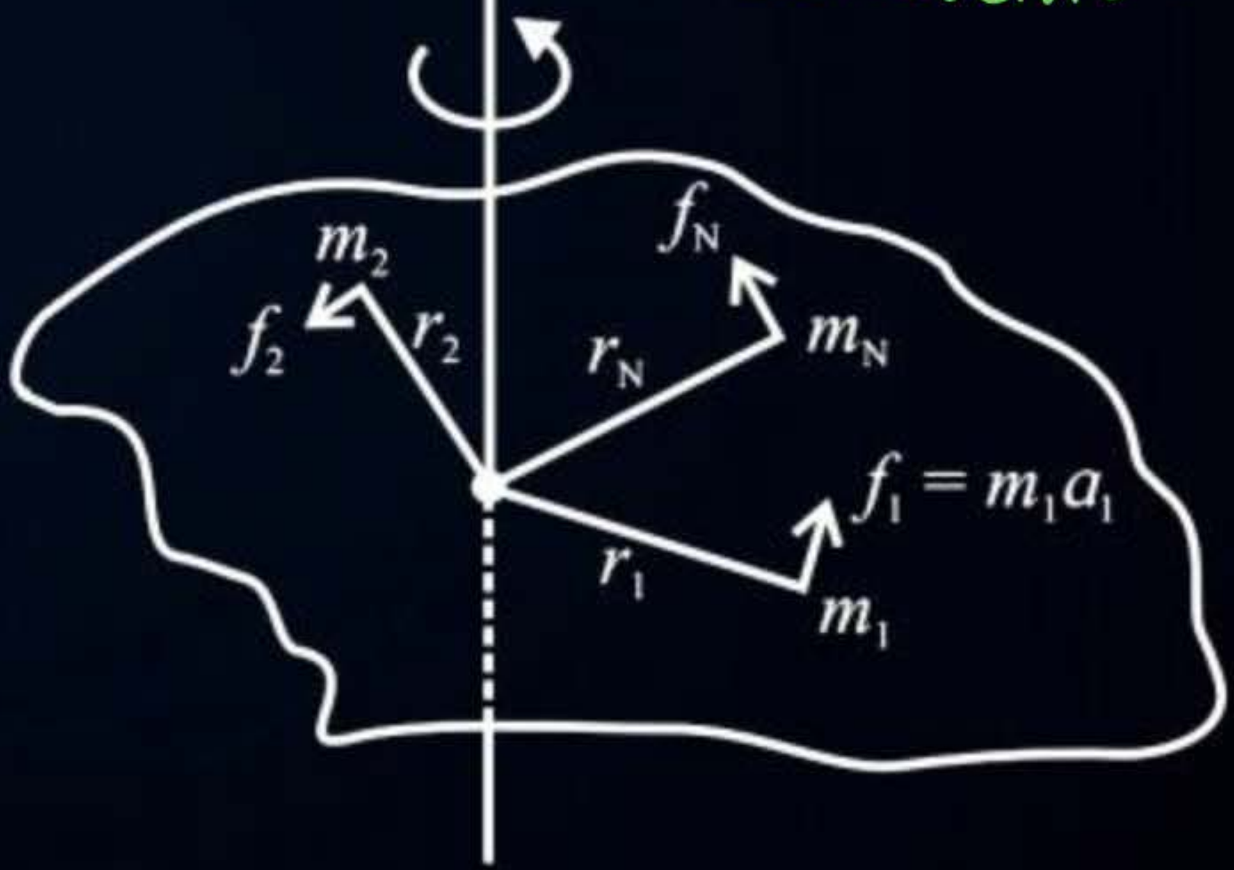
$$\tau_1 = m_1 r_1^2 \alpha \qquad \tau_n = m_n r_n^2 \alpha.$$

$$\tau_2 = m_2 r_2^2 \alpha$$

$$v = r\omega$$

$$a = r\alpha.$$

α = constant.



Expression for torque

$$\tau = \tau_1 + \tau_2 + \ldots + \tau_n$$

$$= (m_1 r_1^2 + m_2 r_2^2 + \ldots + m_n r_n^2)\alpha$$

$$\boxed{\tau = I\alpha}$$

QUESTION

Calculate the moment of inertia of a ring of mass 500 g and radius 0.5 m about an axis of rotation passing through (i) its diameter (ii) a tangent perpendicular to its plane.

(A) 0.25 kg.m^2

(B) 0.30 kg.m^2

(C) 0.40 kg.m^2

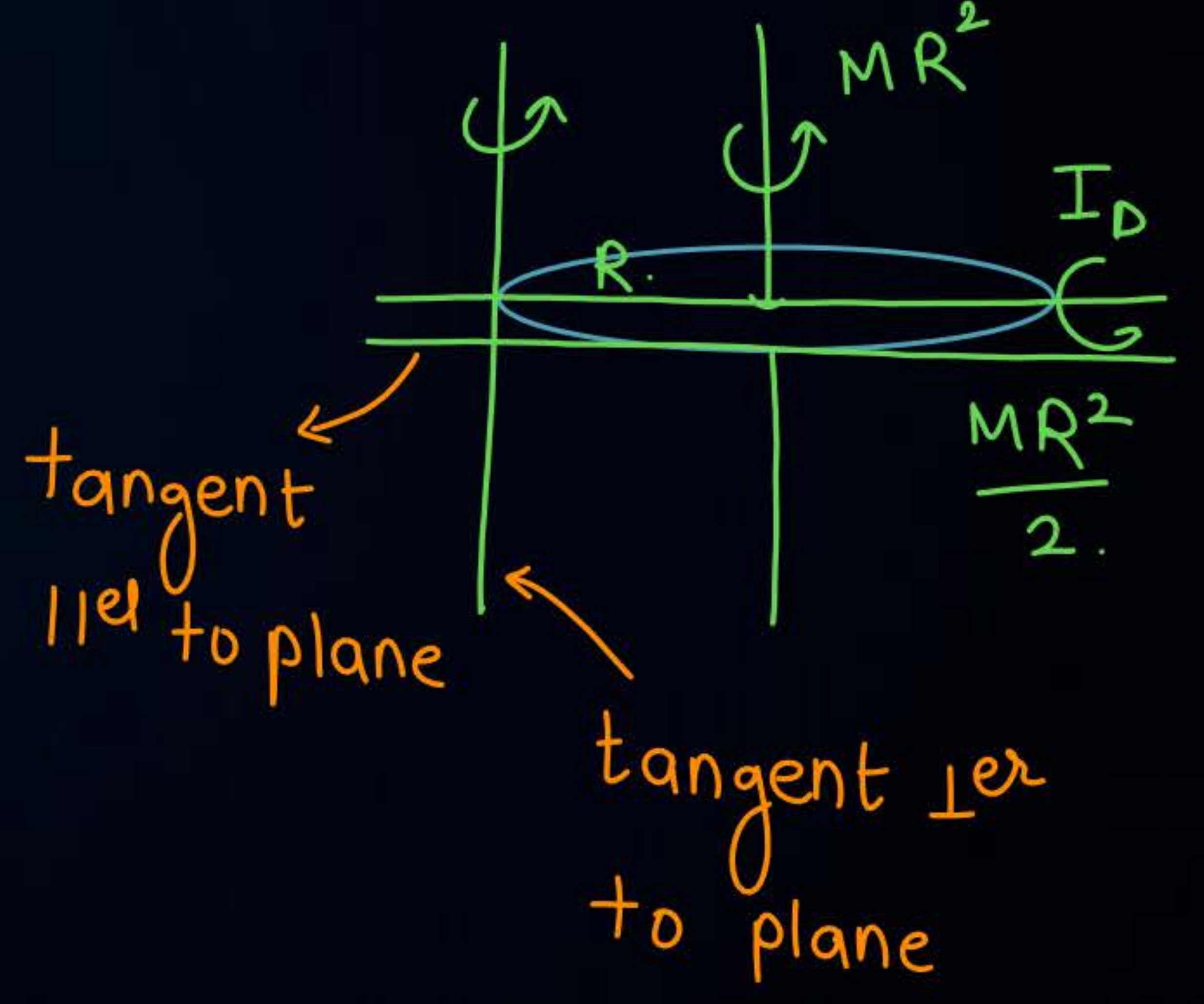
(D) 0.20 kg.m^2

$M = 0.5$ kg

$R = 0.5$ m

i) M of I about diameter

By Perpendicular Axis theorem $I_D = \frac{MR^2}{2}$



$$\therefore I_D = \frac{0.5 \times 0.5 \times 0.5}{2}$$

$$= \frac{125 \times 10^{-3}}{2}$$

$$I_D = 62.5 \times 10^{-3}\ Kgm^2$$

(ii) By Parallel Axis theorem

$$I_0 = I_c + Mh^2$$

$$= MR^2 + MR^2$$

$$I_0 = 2MR^2$$

$$I_0 = 2 \times 0.5 \times 0.5 \times 0.5$$

$$= 250 \times 10^{-3}$$

$$I_0 = 0.25\ Kgm^2$$

QUESTION

A metal ring of mass 1 kg has moment of inertia 1 kg.m^2 for rotation about its diameter. It is melted and recast into a thin uniform disc of the same radius. What will be the disc's moment of inertia when rotated about its own axis?

(A) 1 kg.m^2

(B) 3 kg.m^2

(C) 2 kg.m^2

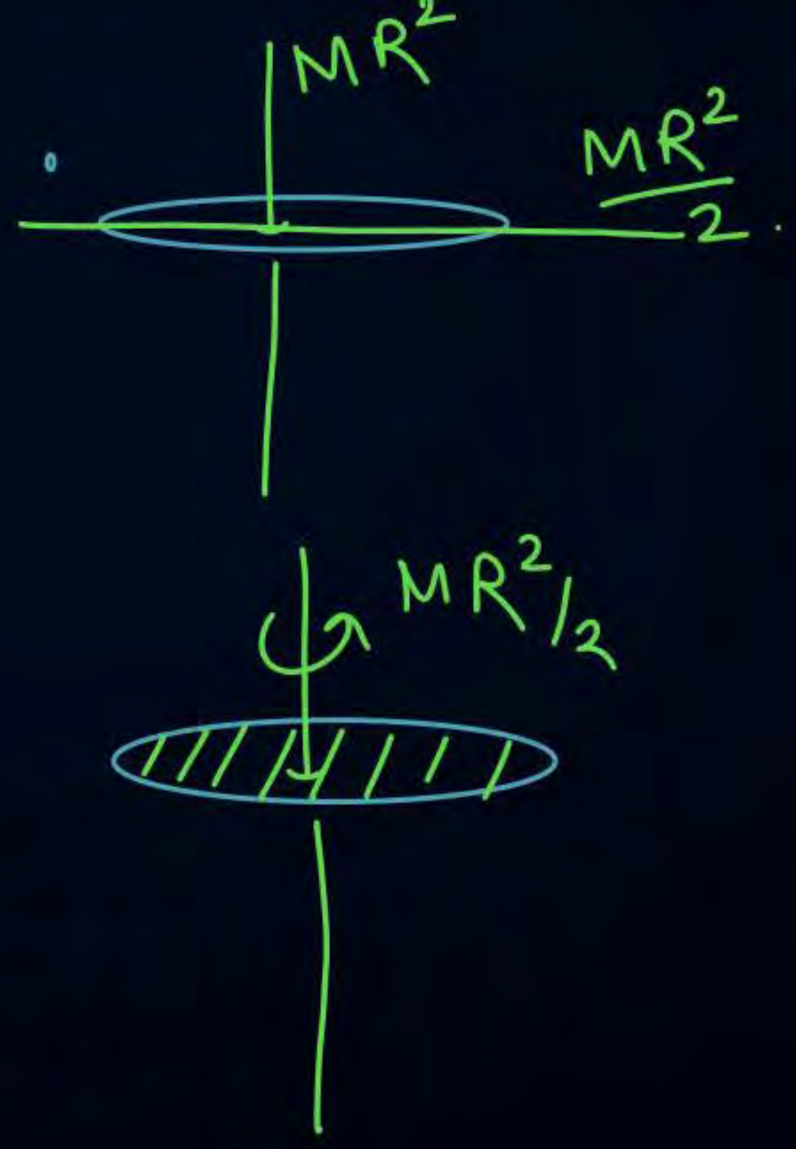
(D) 1.5 kg.m^2

$M_R = 1\,kg$

$I_{R_D} = 1\,kg\,m^2$

$M_R = M_D$.

$R_R = R_D$



Summary

1) $I_o = I_c + Mh^2$

2) $I_z = I_x + I_y$

3) $L = I\omega = rp$

4) $\tau = I\omega = rf$

Homework

1) Memorize statement of ||el & ⊥er Axis theorem

2) Revise all previous notes.

3) Solve all numericals multiple times.

धन्यवाद